# Order and rack allocation in a mobile rack environment – Dantzig-Wolfe decompositions

ENSIIE-CORO

Ana Flavia Macambira

Alain Faye

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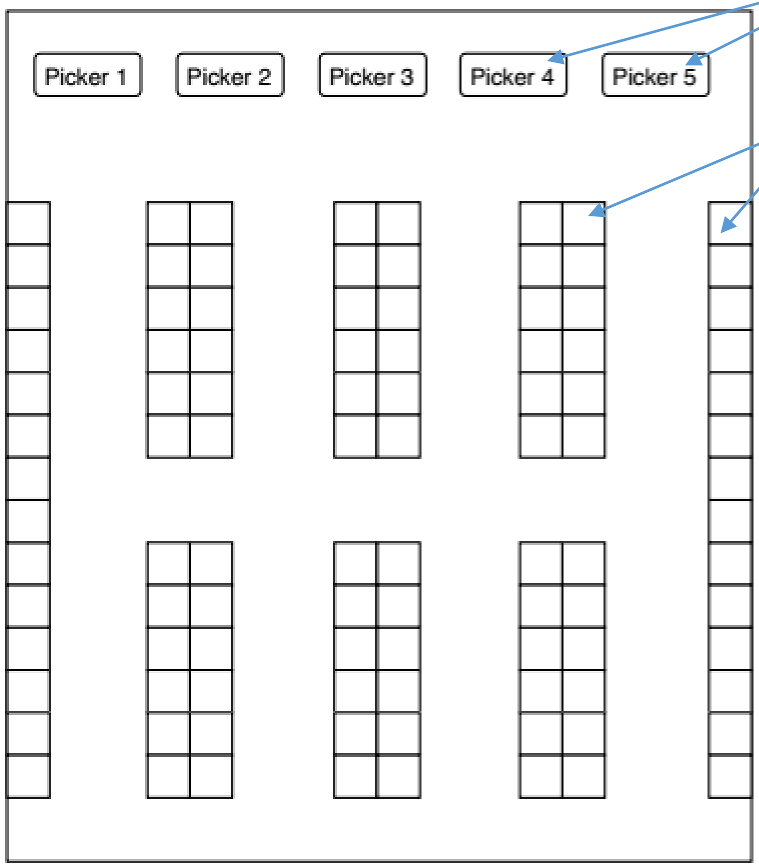
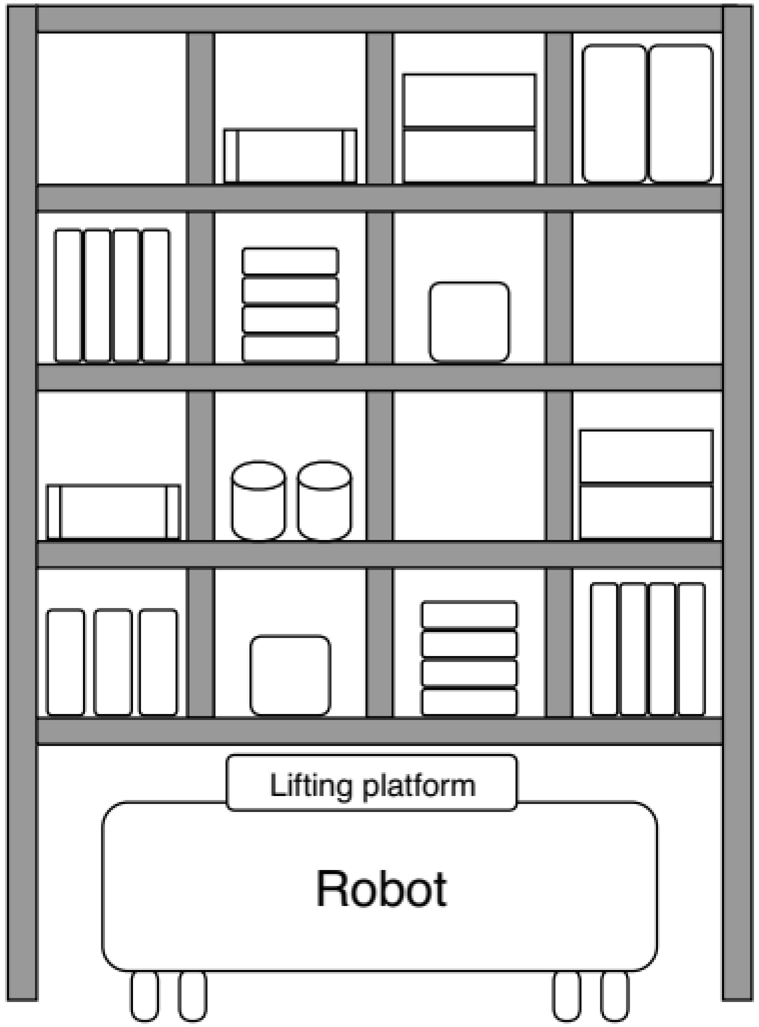
# Centre de distribution – Distribution center




Préparateurs de commande
Pickers

Étagères mobiles
Moveable Racks

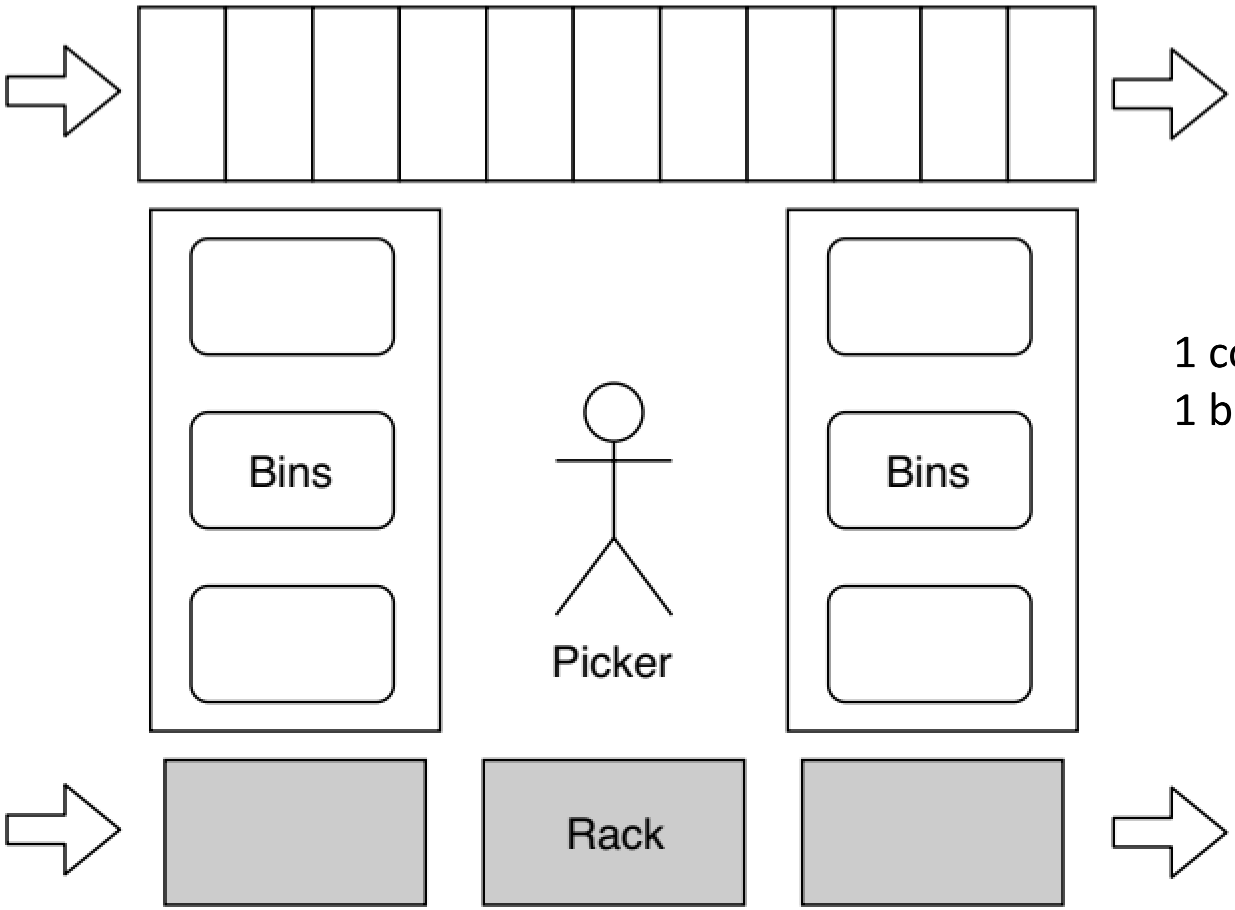
# Etagère mobile – Mobile rack




Une étagère mobile contient
des produits
Moveable rack contains products

Le robot permet de déplacer l'étagère
Mobile robot brings moveable rack

# Préparateur-Picker




Tapis roulant qui emporte les commandes prêtes
Conveyor belt carries away completed orders

1 corbeille = 1 commande
1 bin = 1 order

Arrivée des étagères mobiles
Arrival of moveable racks

# Order Allocation – Allocation des commandes

- Commandes - Orders
- Étagères mobiles - Racks
- Attribuer commandes aux préparateurs – Orders are assigned to pickers
- Attribuer racks aux préparateurs pour qu'ils puissent réaliser les commandes qui leur sont allouées – Racks are assigned to pickers in order to prepare assigned orders
- Un rack est attribué à au plus un préparateur – A rack is assigned to at most one picker
- Une commande est attribuée à au plus un préparateur – An order is assigned to at most one picker

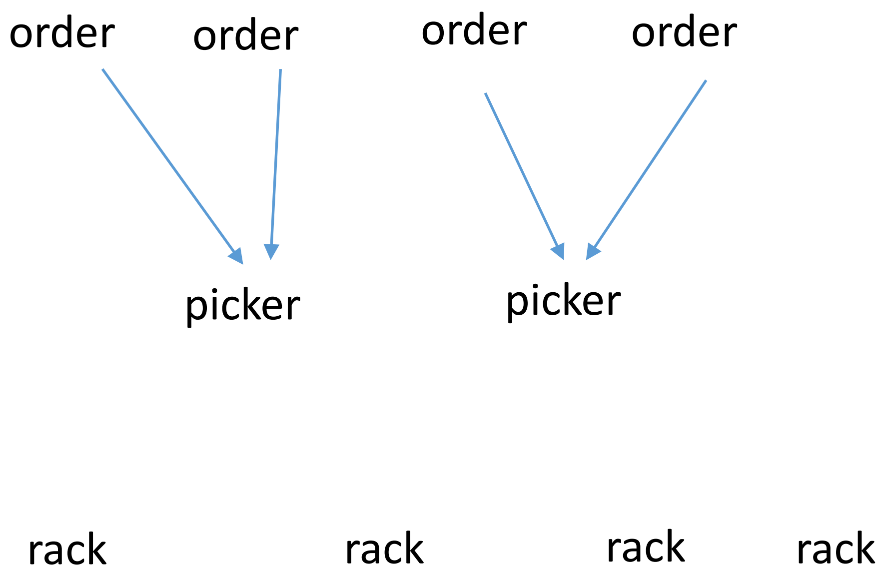
# Commandes (orders)
# Préparateurs (pickers)
# Etagères amovibles (racks)

order order order order

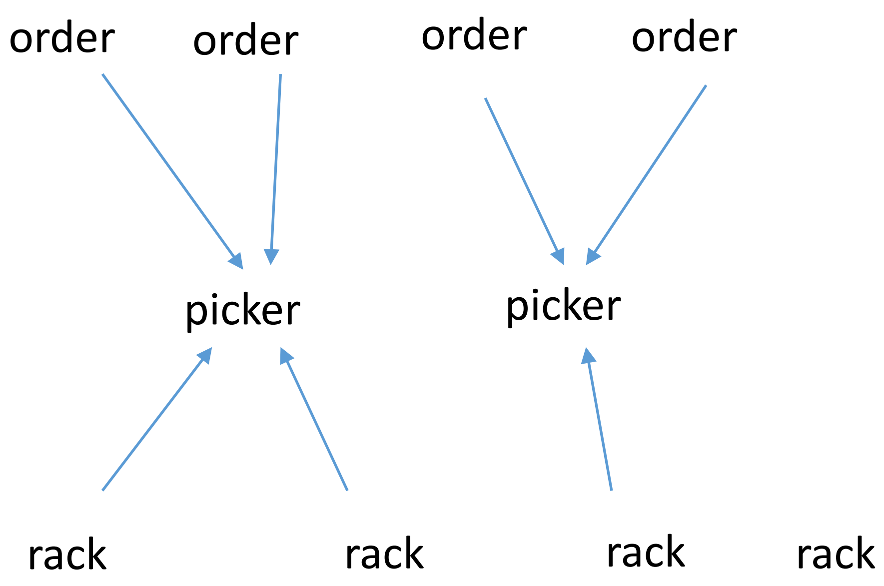
picker picker

rack rack rack rack

# Allocation des commandes aux pickers – Orders are assigned to pickers



# Allocation des racks aux pickers pour apporter les produits commandés dans les ordres – Racks are assigned to pickers in order to prepare assigned orders



# Les données - Data

- N total number of products in the distribution center
- R total number of racks in the distribution center
- O total number of orders
- RS total number of different products in a rack
- $q_{io}$ = units of product *i* in order *o*
- $s_{ir}$ = units of product *i* in rack *r*
- 2 kinds of orders F and S
  - F orders that must be treated now
  - S orders that can be treated now or later

# Fonctions objectifs – Objective functions

- Minimize total number of required racks
- Maximize total number of second orders that are treated
- Optimize a linear combination of these 2 criterions
  - Lexicographic optimization : minimize total number of required racks and next number of second orders that are treated

# Modèle PLNE - MILP Model

## Lexicographic optimization: racks and next second orders

$$\min_{x,y} \sum_{r \in R} (|S| + 1) u_r - \sum_{o \in S} v_o$$

$$\begin{cases} \sum_{p \in P} x_{op} = 1 \ o \in F \quad (1) \\ \sum_{p \in P} x_{op} = v_o \ o \in S \quad (2) \\ \sum_{p \in P} y_{rp} = u_r \ r \in R \quad (3) \\ \sum_{o \in O} x_{op} \leq C_p \ p \in P \quad (4) \\ \sum_{r \in R} s_{ir} y_{rp} \geq \sum_{o \in O} q_{io} x_{op} \ \ p \in P, i \in N \ (5) \\ x_{op}, y_{rp} \in \{0,1\} \\ v_o, u_r \in [0,1] \end{cases}$$

# Lagrangian Relaxation – A short recall

# Primal problem

$(P) \quad \min_{x \in X} f(x)$ subject to $g(x) \leq 0, h(x) = 0$

with $g(x) = \begin{pmatrix} g_1(x) \\ \vdots \\ g_m(x) \end{pmatrix}$ and $h(x) = \begin{pmatrix} h_1(x) \\ \vdots \\ h_p(x) \end{pmatrix}$ and $X \subset \mathbb{R}^n$

# Lagrangian relaxation

- Lagrangian function

$L(x, \lambda, \mu) = f(x) + \lambda \circ g(x) + \mu \circ h(x)$ with $\lambda \in \mathbb{R}_+^m$ and $\mu \in \mathbb{R}^p$

- Dual function

$$\theta(\lambda, \mu) = \min_{x \in X} L(x, \lambda, \mu)$$

- Dual problem

$$\max_{\lambda,\mu} \theta(\lambda, \mu) \text{ subject to } \lambda \geq 0$$

# Weak duality theorem

- For all $x \in X$ s.t. $g(x) \leq 0$ and $h(x) = 0$

  For all $\lambda \in \mathbb{R}_+^m$ and $\mu \in \mathbb{R}^p$

$$f(x) \geq \theta(\lambda, \mu)$$

- Consequently, the dual function provides a lower bound of the optimal value of the primal problem

# Decomposition « simple »

# Decomposition « simple »

Lagrangian relaxation of supply constraints (5)
Dual Variables $\alpha$ are nonnegative

$$
\begin{cases}
\sum_{p \in P} x_{op} = 1 \; o \in F & (1) \\
\sum_{p \in P} x_{op} = v_o \; o \in S & (2) \\
\sum_{p \in P} y_{rp} = u_r \; r \in R & (3) \\
\sum_{o \in O} x_{op} \leq C_p \; p \in P & (4) \\
\sum_{r \in R} s_{ir} y_{rp} \geq \sum_{o \in O} q_{io} x_{op} \;\; p \in P, i \in N \quad \times \boldsymbol{\alpha_{pi}} & (5) \\
x_{op}, y_{rp} \in \{0,1\} & \\
v_o, u_r \in [0,1] &
\end{cases}
$$

The problem is splitted into 2 independent sub-problems: one with variables $x, v$ and the second with variables $y, u$

$$\min_{x} - \sum_{o \in S} v_o + \sum_{o \in O} \sum_{p \in P} x_{op} \sum_{i \in N} \alpha_{pi} q_{io}$$

$s.c.$ (1), (2), (4)

$$\min_{y} \sum_{r \in R} (|S| + 1) u_r + \sum_{r \in R} \sum_{p \in P} y_{rp} \sum_{i \in N} (-\alpha_{pi}) s_{ir}$$

$s.c.$ (3)

# Problème maître – Master Problem

- Variables of the master problem
  - Variables $\lambda^1$ for the sub-problem $x$
  - Variables $\lambda^2$ for the sub-problem $y$
- Constraints of the master problem
  - 2 convexity constraints, one for each sub-problem
  - Supply Constraints (5)

# Les coûts réduits – Reduced costs

- Dual variables of convexity constraints
  - Let $\eta^1$ be the dual variable of convexity constraint of sub-problem $x$
  - Let $\eta^2$ be the dual variable of convexity constraint of sub-problem $y$
- Minimum reduced costs of the variables of the master problem
  - Minimum reduced cost of $\lambda^1$ = value of sub-problem $x - \eta^1$
  - Minimum reduced cost of $\lambda^2$ = value of sub-problem $y - \eta^2$

# Borne inférieure – Lower Bound

- The lower bound is given by the value of the dual function on the current dual variables $\alpha_{pi}$
- Concretely, it is the sum of the values of the 2 sub-problems

# Decomposition according to the pickers

# Décomposition selon les *pickers* - Decomposition according to the pickers

Lagrangian relaxation of coupling constraints (1), (2), (3)
Dual variables α, β are positive or negative

$$
\begin{cases}
\sum_{p\in P} x_{op} = 1 \quad o \in F & \times \boldsymbol{\alpha_o} \ (1) \\
\sum_{p\in P} x_{op} = v_o \quad o \in S & \times \boldsymbol{\alpha_o} \ (2) \\
\sum_{p\in P} y_{rp} = u_r \quad r \in R & \times \boldsymbol{\beta_r} \ (3) \\
\sum_{o\in O} x_{op} \le C_p \quad p \in P & (4) \\
\sum_{r\in R} s_{ir} y_{rp} \ge \sum_{o\in O} q_{io} x_{op} \quad p \in P, i \in N & (5) \\
x_{op}, y_{rp} \in \{0,1\} & \\
v_o, u_r \in [0,1] &
\end{cases}
$$

The problem is splitted into *P* independent sub-problems, one for each picker *p*, with variables $x_{op}, y_{rp}$

$$\min_{x,y} \sum_{r\in R}[(|S|+1)+\beta_r]u_r - \sum_{r\in R} y_{rp}\beta_r - \sum_{o\in S} v_o(1-\alpha_o) + \sum_{o\in F}\alpha_o - \sum_{o\in O} x_{op}\alpha_o$$

$$s.c.\ (4),(5)$$

Here $u_r$ , $v_o$ have constant values fixed by the master problem

$$\sum_{r\in R}[(|S|+1)+\beta_r]u_r - \sum_{o\in S} v_o(1-\alpha_o) + \sum_{o\in F}\alpha_o + \min_{x,y} - \sum_{r\in R} y_{rp}\beta_r - \sum_{o\in O} x_{op}\alpha_o$$

$$s.c.\ (4),(5)$$

# Problème maître – Master problem

- Variables of the master problem
  - $\lambda^p$ for all picker $p = 1, \dots, p$
- Constraints of the master problem
  - $P$ convexity constraints, one for each picker $p$
  - Constraints (1), (2), (3) previously relaxed

# Les coûts réduits – Reduced costs

- Dual variables of the convexity constraints
  - Let $\eta^p$ be the dual variable relative to the convexity constraint of picker $p$
- Minimum reduced costs of the variables of the master problem
  - Minimum reduced cost of variable $\lambda^p$

$$\min_{\substack{x,y \\ \text{s.c. } (4),(5)}} \; -\sum_{r \in R} y_{rp}\beta_r - \sum_{o \in O} x_{op}\alpha_o - \eta^p$$

# Borne inférieure – Lower bound

- The lower bound is equal to the value of the dual fonction at the current dual variables $\beta_r$ and $\alpha_o$
- Concretely, it is equal to the sum of sub-problem values over pickers $p = 1, \ldots, P$

# Travail proposé – What you have to do

- Implementation of MILP model
- Study of a Dantzig-Wolfe decomposition
  - Lower bound, comparison with MILP
  - Build a feasible integer solution from the last master problem , comparison with MILP
- Coding in Julia-JuMP

# Les instances de tests – Tests instances

- web4.ensiie.fr/~faye/AnaFlavia/
- Instances are in text files
- Code Julia **lecture.jl** in order to read the instances
- lecture.jl initializes struct « *donnees* »
  - N, R, O, RS, P are integers
  - S, Q are arrays with 2 dimensions
  - Capa, FO, SO are vectors

# La structure *donnees* – Struct *donnees*

| N | R | O | RS | P | S | Q | Capa | FO | SO |
|---|---|---|---|---|---|---|---|---|---|
| nombre de produits | nombre de racks | nombre d'ordres | nombre de produits distincts dans un rack | nombre de pickers | matrice produits × racks S[i][r]=quantité de produits i dans rack r | matrice produit × ordre Q[i][o]=quantité de produit i dans l'ordre o | vecteur indexé par les pickers Capa[p]=capacité du picker p | vecteur des first orders FO[n]=first order en position n 1≤n≤nombre de FO | vecteur des second orders SO[n]=second order en position n 1≤n≤nombre de SO |
| Donnée spécifiée dans le fichier d'instance | Donnée spécifiée dans le fichier d'instance | Donnée spécifiée dans le fichier d'instance | Donnée spécifiée dans le fichier d'instance | Donnée à définir, le champ est à remplir | Donnée spécifiée dans le fichier d'instance | Donnée spécifiée dans le fichier d'instance | Donnée à définir, le champ est à remplir | Donnée à définir, le champ est à remplir | Donnée à définir, le champ est à remplir |

# Restitution des travaux – Restitution of work

- Travail en binôme
- Suivi le long des séances
- Rapport écrit à rendre
  - Number of generated columns – CPU time
  - Lower Bounds
  - Comparison with MILP
  - Evolution of the Lower Bound and Upper Bound (value of master problem) along the iterations of column generation algorithm

# Bibliography


- Order allocation, rack allocation and rack sequencing for pickers in a mobile rack environment. Cristiano A Valle Universidade Federal de Minas Gerais, John E Beasley Brunel University London (2019).
- Dualité lagrangienne. Alain Faye http://web4.ensiie.fr/~faye/mom/MOM_cours_TD/MOM_5/Dualite_lagrangienne.pdf