Kindly Bent To Free Us

Gabriel Radanne Peter Thiemann December 6, 2018

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let fd : Tls_lwt.Unit.t = .....
let input, output = Tls_lwt.of_t fd
... (* read some things *)
let%lwt () = Lwt_io.close input in
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- Closures
- Monads
- Existential types
- ...

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Many places in OCaml where enforcing linearity is useful:

- IO (File handle, channels, network connections, ...)
- Protocols (With session types! Mirage libraries)
- One-shot continuations (effects!)
- Transient data-structures
- C-style "struct parsing"
- ...

Goals:

- Complete and principal type inference
- Impure strict context
- Works well with type abstraction
- Play balls with various other ongoing works (Effects, Resource polymorphism, ...)

Non Goals:

- Support every linear code pattern under the sun
- Design associated compiler optimisations/GC integration (yet)

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The Affe language

Types and Behaviors

In Affe, the behavior of a variable is determined by its type:
type channel : A (* channel is Affine! *)

```
let with_file s f =
 let c = open_channel s
 let c = f c in
val with_file : string -> (channel -> channel)
let () =
 let r = ref None in
    (fun c -> r := Some c ; c) (* ¥ No! *)
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 let r = ref None in
  with_file "thing"
    (fun c -> r := Some c ; c) (* ¥ No! *)
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Infer unrestricted in case of duplication:

let f = fun c -> r := Some c ; c
val f : ('a : U) . 'a -> 'a

So far, two kinds:

A Affine types: can be used at most onceU Unrestricted types

Additionally, we have:

 $\mathbf{U} \leq \mathbf{A}$

What about closures?

let f = fun a -> fun b -> (a, b)
val f : 'a -> 'b -> 'a * 'b (* ? *)

What about closures?

let f = fun a -> fun b -> (a, b)
val f : ('a : 'k) => 'a -> 'b -{'k}> 'a * 'b

You can annotate the kinds on type declarations. Vanilla OCaml references are fully unrestricted:

type ('a : U) **ref** : U = ...

We can also have constraints on kinds. The pair type operator: **type** * : (k1 < k) & (k2 < k) => k1 -> k2 -> k You can annotate the kinds on type declarations. Vanilla OCaml references are fully unrestricted:

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type * : $(k1 < k) & (k2 < k) \implies k1 \implies k2 \implies k$

Mixing with abstraction:

```
module AffineArray : sig
  type -'a w : A
  val create :
    ('a : U) . int -> 'a -> 'a w
  val set : 'a w -> int -{A}> 'a -{A}> 'a w
  type +'a r : U
  val freeze : 'a w -> 'a r
  val get : int -> 'a r -> 'a
end
```

The calculus

Expressions

$$e ::= c | x | (e e') | \lambda x.e$$

| let $x = e$ in e'
| (K e) | elim_K e

Type Expressions
$$\tau ::= \alpha \mid \tau \xrightarrow{k} \tau \mid (\tau^*) t$$
$$k ::= \kappa \mid \ell \in \mathcal{L}$$

-



Constraints are only acceptable in schemes:

$$\sigma ::= \forall \kappa^* \forall (\alpha : k)^* . (C \Rightarrow \tau)$$
$$\theta ::= \forall \kappa^* . (C \Rightarrow k_i^* \to k)$$

The constraint language in schemes is limited to list of inequalities:

$$C ::= (k \le k')^*$$



Variables can be kind-polymorphic and all their instances might not have the same kinds.

 \implies We must track the kinds of all use-sites for each variable.

Use maps (Σ) associates variables to multisets of kinds and are equipped with three operations:

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- $\Sigma_1 | (C_1, \psi_1) | \Gamma \vdash_w e_1 : \tau_1$
- $\Sigma_2 | (C_2, \psi_2) | \Gamma \vdash_{\mathsf{w}} e_2 : \tau_2$
- Add $(\Sigma_1\cap\Sigma_2\leq {\sf U})$ to the constraints
- . . .
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A slightly more general context: $\mathcal{C}_{\mathcal{L}}$ is the constraint system:

$$C ::= (\tau_1 \leq \tau_2) \mid (k_1 \leq k_2) \mid C_1 \land C_2 \mid \exists \alpha. C$$

where $k ::= \kappa \mid \ell \in \mathcal{L}$ and $(\mathcal{L}, \leq_{\mathcal{L}})$ is a complete lattice.

Respect, among other things:

$$\frac{\ell \leq_{\mathcal{L}} \ell'}{\vdash_{\mathsf{e}} (\ell \leq \ell')} \qquad \qquad \vdash_{\mathsf{e}} (k \leq \ell^{\top}) \qquad \qquad \vdash_{\mathsf{e}} (\ell^{\perp} \leq k)$$

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Example : $\lambda f \cdot \lambda x \cdot ((f x), x)$

Raw constraints:

$$(\alpha_f : \kappa_f)(\alpha_x : \kappa_x) \dots$$
$$(\alpha_f \le \gamma \xrightarrow{\kappa_1} \beta) \land (\gamma \le \alpha_x) \land (\beta * \alpha_x \le \alpha_r) \land (\kappa_x \le \mathsf{U})$$

We unify the types and discover new constraints:

$$\alpha_r = (\gamma \xrightarrow{\kappa_3} \beta) \xrightarrow{\kappa_2} \gamma \xrightarrow{\kappa_1} \beta * \gamma$$
$$(\kappa_x \le \mathsf{U}) \land (\kappa_\gamma \le \kappa_x) \land (\kappa_x \le \kappa_r) \land (\kappa_\beta \le \kappa_r) \land (\kappa_3 \le \kappa_f) \land (\kappa_f \le \kappa_1)$$

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Normalization is complete and principal.

$$\lambda f.\lambda x.((f \ x), x):$$
$$\forall \kappa_{\beta}\kappa_{1}\kappa_{2}\kappa_{3}(\gamma: \mathsf{U})(\beta:\kappa_{\beta}). \ (\kappa_{3} \leq \kappa_{1}) \Rightarrow (\gamma \xrightarrow{\kappa_{3}} \beta) \xrightarrow{\kappa_{2}} \gamma \xrightarrow{\kappa_{1}} \beta * \gamma$$

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$$\forall \kappa_{\beta} \kappa(\gamma : \mathsf{U})(\beta : \kappa_{\beta}) . (\gamma \xrightarrow{\kappa} \beta) \to \gamma \xrightarrow{\kappa} \beta * \gamma$$

$$\implies$$
 Unfinished, need to investigate principality

The only requirement is that $\ell^{\perp} = \mathbf{U}$.

- A doesn't appear in the typing rules. It only comes from the buitins and/or the type declarations.
- The lattice doesn't have to be finite.
- The constraint language can be expanded further.

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Conclusion

I presented a somewhat minimalistic approach to add linear types to an existing ML language (like OCaml).

- Based on kinds and constraints
- Works with type abstraction and modules
- Support type inference
- Doesn't break the whole ecosystem

The system is still small. We must look at concrete code pattern used in OCaml and decide how to support them.

Area of future work:

- Explore various interactions with modules
- Borrowing
- Better control-flow interactions

Close(Talk)

- Qualified types are coming for modular implicits anyway.
- Having proper kinds would fix many weirdness (rows, ...) and enable nice extensions (units of measures).
- \bullet I could make Eliom even better with them! \odot

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Going further

- 1. Richer type system
- 2. Modules
- 3. Borrowing
- 4. Prototype cool APIs with it

Constraints in a similar style have been applied to:

- (Relaxed) value restriction
- GADTs
- Rows
- Type elaboration
- . . .

- Type abstraction
- Linear/affine values in modules
- Functors
- Separate compilation

● Type abstraction ✔

Can declare unrestricted types and expose them as Affine.

- Linear/affine values in modules
- Functors
- Separate compilation

- Type abstraction
- Linear/affine values in modules

Behave like tuples: take the LUB of the kinds of the exposed values.

What about values that are not exposed? They don't matter!

- Functors
- Separate compilation

- Type abstraction
- Linear/affine values in modules
- Functors

What happens if a functor takes a module containing affine values?

 \implies We need kind annotation on the functor arrow. . .

• Separate compilation
Several distinct problematic:

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What about linear/affine constants?

 \implies Should probably be forbidden...

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What about linear/affine constants?

 \implies Should probably be forbidden...

But what about stdout ?

Borrowing seem essential to express many patterns found in OCaml.

Read-only borrows, in CCHashTrie:

val add_mut : id -> key -> 'a -> 'a t -> 'a t
(* add_mut ~id k v m behaves like add k v m, except
 it will mutate in place whenever possible. *)

Mutable borrows, in lacaml:

val Lacaml.D.sycon :

... -> ?iwork:Common.int32_vec -> mat -> float
(* iwork is an optional preallocated work buffer *)

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Borrowing

"Resource Polymorphism" has the following lattice:



It would requires:

- More syntactic annotations
- Regions

- Ownership approaches
- Capabilities and typestates
- Substructural type systems
- ...

• Ownership approaches

Suitable to imperative languages (Rust, ...).

- Capabilities and typestates
- Substructural type systems
- . . .

- Ownership approaches
- Capabilities and typestates

Often use in Object-Oriented contexts (Wyvern, Plaid, Hopkins Objects Group, ...).

- Substructural type systems
- ...

- Ownership approaches
- Capabilities and typestates
- Substructural type systems

Many variations, mostly in functional languages:

- Inspired directly from linear logic (Linear Haskell, Walker, ...)
- Uniqueness (Clean)
- Kinds (Alms, Clean, F°)
- Constraints (Quill)

• . . .

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- . . .

Mix of everything: Mezzo

- Ownership approaches
- Capabilities and typestates
- Substructural type systems
- ...

HM(X) (Odersky et al., 1999) is a framework to build an HM type system (with inference) based on a given constraint system. We provide two additions:

- A small extension of HM(X) that tracks kinds and linearity
- An appropriate constraint system

References

Martin Odersky, Martin Sulzmann, and Martin Wehr. 1999. Type Inference with Constrained Types. *TAPOS* 5, 1 (1999), 35–55.